REASONING AND PROOFS

Geometry

Chapter 2

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Geometry*
  - *By Larson and Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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2.1 CONDITIONAL STATEMENTS

Objectives: By the end of the lesson,

- I can write conditional statements.
- I can write biconditional statements.

2.1 CONDITIONAL STATEMENTS

- Determine whether each conditional statement is true or false. Justify your answer.
- **i.** If yesterday was Wednesday, then today is Thursday.
- **ii.** If an angle is acute, then it has a measure of 30°.
- **iii.** If a month has 30 days, then it is June.
- **iv.** If $\triangle ADC$ is a right triangle, then the Pythagorean Theorem is valid for $\triangle ADC$.
- **v.** If a polygon is a quadrilateral, then the sum of its angle measures is 180°.
- **vi.** If points A, B, and C are collinear, then they lie on the same line.

2.1 CONDITIONAL STATEMENTS

Conditional Statement

- Logical statement with two parts
  - Hypothesis
  - Conclusion
- Often written in If-Then form
  - If part contains hypothesis
  - Then part contains conclusion

If we confess our sins, then He is faithful and just to forgive us our sins. 1 John 1:9

Red is hypothesis, Gray is conclusion

2.1 CONDITIONAL STATEMENTS

If-then statements

$p \rightarrow q$

The if part implies that the then part will happen.

The then part does NOT imply that the first part happened.

If you are hungry, then you should eat.
John is hungry, so…
Megan should eat, so…

John is hungry, so… he should eat.
Megan should eat, so… no conclusion. This is backwards. She could be diabetic and need to eat to get her blood sugar level correct.

2.1 CONDITIONAL STATEMENTS

Negation

~p

Turn it to the opposite.

- Example:
  - The board is white.

The board is not white.

2.1 CONDITIONAL STATEMENTS

Converse

q → p

Switch the hypothesis and conclusion

- Example:
  - **If we confess our sins, then he is faithful and just to forgive us our sins.**
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Converse = If he is faithful and just to forgive us our sins, then we confess our sins.
  - Does not necessarily make a true statement
    (He may be faithful and just, but many people still don't ask for forgiveness.)

2.1 CONDITIONAL STATEMENTS

Inverse

$\sim p \rightarrow \sim q$

Negating both the hypothesis and conclusion

- Example:
  - **If we confess our sins, then he is faithful and just to forgive us our sins.**
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Inverse = If we don't confess our sins, then he is not faithful and just to forgive us our sins.
  - Not necessarily true (He is still faithful and just even if we do not confess.)

2.1 CONDITIONAL STATEMENTS

Contrapositive

$\sim q \rightarrow \sim p$

Take the converse of the inverse

- Example:
  - If we confess our sins, then he is faithful and just to forgive us our sins.
    - p = we confess our sins
    - q = he is faithful and just to forgive us our sins
  - Contrapositive (inverse of converse) = If he is not faithful and just to forgive us our sins, then we won't confess our sins.
  - Always true.

2.1 CONDITIONAL STATEMENTS

- Write the following in If-Then form and then write the converse, inverse, and contrapositive
  - All whales are mammals.

If-Then: If it is a whale, then it is a mammal.
Converse: If it is a mammal, then it is a whale.
Inverse: If it is not a whale, then it is not a mammal.
Contrapositive: if it is not a mammal, then it is not a whale.

2.1 CONDITIONAL STATEMENTS

Biconditional Statement

- Logical statement where the if-then and converse are both true
- Written with “if and only if”
  - iff

An angle is a right angle if and only if it measure 90°.

2.1 CONDITIONAL STATEMENTS

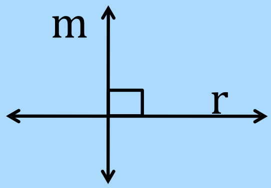
- All definitions can be written as if-then and biconditional statements

Perpendicular Lines

Lines that intersect to form right angles

m ⊥ r

- Rewrite the definition as a biconditional.

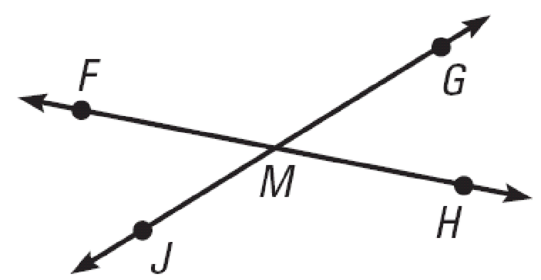


If-then: If lines intersect to form right angles, then they are perpendicular.
Biconditional: Lines are perpendicular iff they intersect to form right angles.

2.1 CONDITIONAL STATEMENTS

- Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.

1. ∠JMF and ∠FMG are supplementary
2. Point M is the midpoint of $\overline{FH}$
3. ∠JMF and ∠HMG are vertical angles.
4. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$



69 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 24, 26, 28, 30, 32, 49, 68, 71, 74, 76

1. True, linear pairs are supplementary
2. False, no information given
3. True, intersecting lines form vertical angles
4. False, no information given

2.2A INDUCTIVE REASONING

Objectives: By the end of the lesson,

- I can use inductive reasoning to make conjectures.

2.2A INDUCTIVE REASONING

- Geometry, and much of math and science, was developed by people recognizing patterns
- We are going to use patterns to make predictions this lesson

2.2A INDUCTIVE REASONING

Conjecture

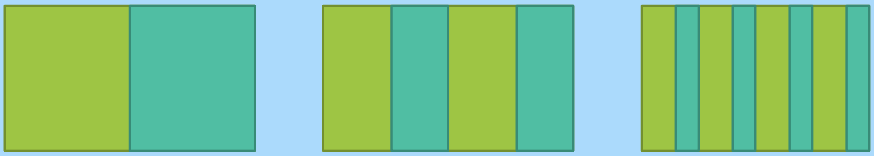
- Unproven statement based on observation

Inductive Reasoning

- First find a pattern in specific cases
- Second write a conjecture for the general case

2.2A INDUCTIVE REASONING

- Sketch the fourth figure in the pattern

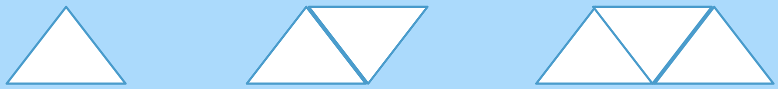


- Describe the pattern in the numbers 1000, 500, 250, 125, … and write the next three numbers in the pattern

Each number is ½ the previous number: 62.5, 31.25, 15.625

2.2A INDUCTIVE REASONING

- Given the pattern of triangles below, make a conjecture about the number of segments in a similar diagram with 5 triangles



- Make and test a conjecture about the product of any two odd numbers

Each figure has two more segments
Third figure has seven segment, so 5^{th} has 7 + 2 + 2 = 11

Product means multiply
Try several: 3(5) = 15; 7(11) = 77; 9(3) = 27
Looks like the product of two odd numbers is **odd**

2.2A INDUCTIVE REASONING

- The only way to show that a conjecture is true is to show **all** cases

- To show a conjecture is false is to show **one** case where it is false
  - This case is called a **counterexample**

2.2A INDUCTIVE REASONING

- Find a counterexample to show that the following conjecture is false
  - The value of x^2 is always greater than the value of x

- 78 #1, 2, 4, 6, 7, 8, 10, 12, 13, 14, 36, 43, 45, 46, 49

Sample answer: let $x = \frac{1}{2}$; $x^2 = \frac{1}{4}$

2.2B DEDUCTIVE REASONING

Objectives: By the end of the lesson,

- I can use deductive reasoning to verify conjectures.
- I can distinguish between inductive and deductive reasoning.

2.2B DEDUCTIVE REASONING

Deductive Reasoning

Use facts, definitions, properties, laws of logic to form an argument.

- Deductive reasoning
  - Always true
  - General → specific
- Inductive reasoning
  - Sometimes true
  - Specific → general

2.2B DEDUCTIVE REASONING

Law of Detachment

If the hypothesis of a true conditional statement is true, then the conclusion is also true.

Detach means comes apart, so the 1st statement is taken apart.

- Example:
  1. **If we confess our sins, then He is faithful and just to forgive us our sins.** 1 John 1:9
  2. Jonny confesses his sins
  3. God is faithful and just to forgive Jonny his sins

2.2B DEDUCTIVE REASONING

1. If you love me, keep my commandments.
2. I love God.
3. ______________________________

1. If you love me, keep my commandments.
2. I keep all the commandments.
3. ______________________________

I keep the commandments

Not Valid

2.2B DEDUCTIVE REASONING

Law of Syllogism

If hypothesis p, then conclusion q.
If hypothesis q, then conclusion r.

If these statement are true,

If hypothesis p, then conclusion r.

then this statement is true

1. If we confess our sins, **He is faithful and just to forgive us our sins.**
2. **If He is faithful and just to forgive us our sins**, then we are blameless.
3. If we confess our sins, then we are blameless.

2.2B DEDUCTIVE REASONING

- If you love me, keep my commandments.
- If you keep my commandments, you will be happy.
- ______________________________

- If you love me, keep my commandments.
- If you love me, then you will pray.
- ______________________________
- 78 #16, 17, 18, 19, 21, 22, 24, 25, 26, 30, 32, 34, 40, 51, 54

2.3 POSTULATES AND DIAGRAMS

Objectives: By the end of the lesson,

- I can identify postulates represented by diagrams.
- I can sketch a diagram given a verbal description.
- I can interpret a diagram.

2.3 POSTULATES AND DIAGRAMS

Postulates (axioms)

Rules that are accepted without proof (assumed)

Theorem

Rules that are accepted only with proof

2.3 POSTULATES AND DIAGRAMS

Basic Postulates (Memorize for quiz!)

- Through any two points there exists exactly one line.
- A line contains at least two points.
- If two lines intersect, then their intersection is exactly one point.
- Through any three noncollinear points there exists exactly one plane.

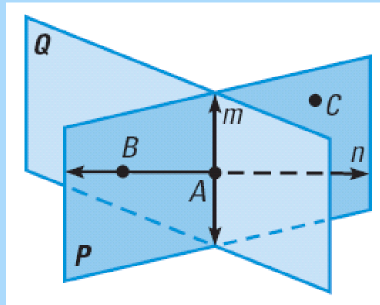
2.3 POSTULATES AND DIAGRAMS

Basic Postulates (continued)

- A plane contains at least three noncollinear points.
- If two points lie in a plane, then the line containing them lies in the plane.
- If two planes intersect, then their intersection is a line.

2.3 POSTULATES AND DIAGRAMS

- Which postulate allows you to say that the intersection of plane *P* and plane *Q* is a line?



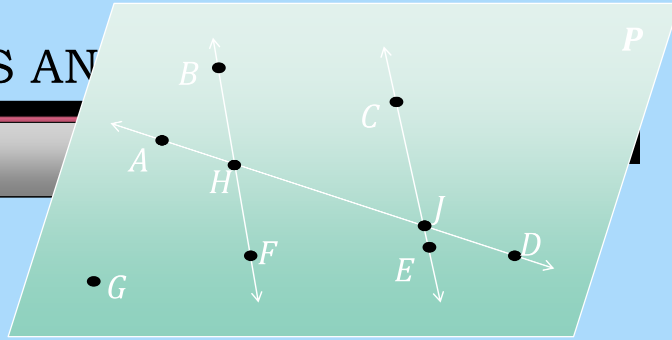
- Use the diagram to write examples of the 1st three postulates from this lesson.

If two planes intersect, then their intersection is a line.

Line *n* passes through points A and B.
Line n contains points A and B
Line m and line n intersect at point A

2.3 POSTULATES AN

Interpreting a Diagram



You can Assume

- All points shown are coplanar
- $\angle AHB$ and $\angle BHD$ are a linear pair
- $\angle AHF$ and $\angle BHD$ are vertical angles
- A, H, J, and D are collinear
- $\overleftrightarrow{AD}$ and $\overleftrightarrow{BF}$ intersect at H

You cannot Assume

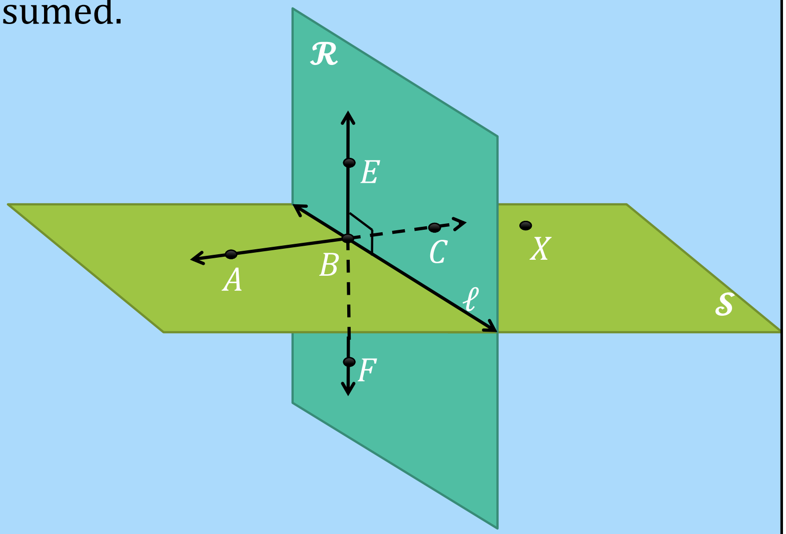
- G, F, and E are collinear
- $\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ intersect
- $\overleftrightarrow{BF}$ and $\overleftrightarrow{CE}$ do not intersect
- $\angle BHA \cong \angle CJA$
- $\overleftrightarrow{AD} \perp \overleftrightarrow{BF}$
- $m\angle AHB = 90°$

2.3 POSTULATES AND DIAGRAMS

- Sketch a diagram showing $\acute{FH} \perp EG$ at its midpoint M.

2.3 POSTULATES AND DIAGRAMS

- Which of the follow cannot be assumed.



- A, B, and C are collinear
- $\overleftrightarrow{EF} \perp$ line ℓ
- $\overleftrightarrow{BC} \perp$ plane $\mathcal{R}$
- $\overleftrightarrow{EF}$ intersects $\overleftrightarrow{AC}$ at B
- line $\ell \perp \overleftrightarrow{AB}$
- Points B, C, and X are collinear

85 #2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 21, 22, 23, 25, 26, 31, 32, 36, 38, 39

BC ⊥ plane R
line ℓ ⊥ AB
Points B, C, and X are collinear

2.4 ALGEBRAIC REASONING

Objectives: By the end of the lesson,

- I can identify algebraic properties of equality.
- I can use algebraic properties of equality to solve equations.
- I can use properties of equality to solve for geometric measures.

2.4 ALGEBRAIC REASONING

- When you solve an algebra equation, you use properties of algebra to justify each step.
- Segment length and angle measure are real numbers just like variables, so you can solve equations from geometry using properties from algebra to justify each step.

2.4 ALGEBRAIC REASONING	
Property of Equality	Numbers
Reflexive	$a = a$
Symmetric	$a = b$, then $b = a$
Transitive	$a = b$ and $b = c$, then $a = c$
Add and Subtract	If $a = b$, then $a + c = b + c$
Multiply and divide	If $a = b$, then $a \cdot c = b \cdot c$
Substitution	If $a = b$, then a may be replaced by b in any equation or expression
Distributive	$a(b + c) = ab + ac$

2.4 ALGEBRAIC REASONING

- Name the property of equality the statement illustrates.
  - If m$\angle 6$ = m$\angle 7$, then m$\angle 7$ = m$\angle 6$.
  - If $JK = KL$ and $KL = 12$, then $JK = 12$.
  - m$\angle W$ = m$\angle W$

Symmetric
Transitive
Reflexive

2.4 ALGEBRAIC REASONING

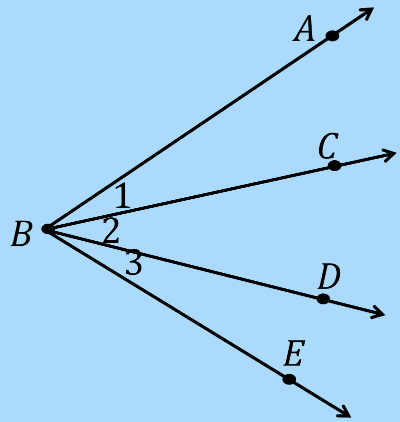
- Solve the equation and write a reason for each step
  - $14x + 3(7 - x) = -1$
- Solve $A = \frac{1}{2} bh$ for b.

$14x + 21 - 3x = -1$	distributive
$11x + 21 = -1$	definition of add (optional step)
$11x = -22$	subtraction
$x = -2$	division

$A = \frac{1}{2} bh$	
$2A = bh$	multiplication
$2A/h = b$	division
$b = 2A/h$	symmetric

2.4 ALGEBRAIC REASONING

- Given: m∠ABD = m∠CBE
- Show that m∠1 = m∠3



- 92 #2, 4, 6, 8, 10, 16, 20, 22, 24, 28, 30, 32, 34, 36, 38, 53, 54, 60, 61, 63

m∠ABD = m∠CBE	(given)
m∠ABD = m∠1 + m∠2	(angle addition post.)
m∠CBE = m∠2 + m∠3	(angle addition post.)
m∠1 + m∠2 = m∠2 + m∠3	(substitution)
m∠1 = m∠3	(subtraction)

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

Objectives: By the end of the lesson,

- I can explain the structure of a two-column proof.
- I can write a two-column proof.
- I can identify properties of congruence.

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

- Pay attention today, we are going to talk about how to write proofs.
- Proofs are like making a peanut butter and jelly sandwich.
- Given: Loaf of bread, jar of peanut butter, and jelly sitting on counter
- Prove: Make a peanut butter and jelly sandwich

Ask: What do we do now? (write down ideas generated on the board)
Ask: Is there any special order for this? (yes, there is and have students start to put the steps in order)
If a student wants a step, such as spread peanut butter on the bread, respond by asking where the peanut butter came from or what are you using. You cannot use an object until you get it.
Write the steps in the first column with justifications for each step in the second column

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

Congruence of segments and angles is reflexive, symmetric, and transitive.

- Writing proofs follow the same step as the sandwich.
  1. Write the given and prove written at the top for reference
  2. Start with the given as step 1
  3. The steps need to be in an logical order
  4. You cannot use an object without it being in the problem
  5. Remember the hypothesis states the object you are working with, the conclusion states what you are doing with it
  6. If you get stuck ask, “Okay, now I have ______. What do I know about ______ ?” and look at the hypotheses of your theorems, definitions, and properties.

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

Complete the proof by justifying each

P **Q** **S**

Given: Points P, Q, and S are collinear

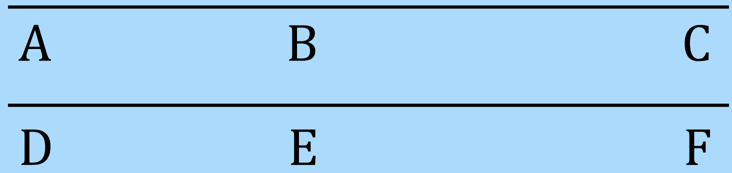
Prove: PQ = PS – QS

Statements	Reasons
Points P, Q, and S are collinear	*Given*
PS = PQ + QS	*Segment addition post*
PS – QS = PQ	*Subtraction*
PQ = PS – QS	*Symmetric*

Students are to come up with reasons

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

Write a two column proof



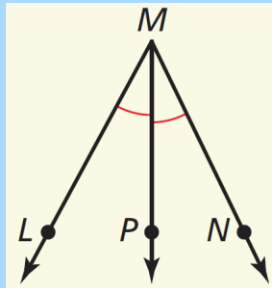
Given: $\overline{AC} \cong \overline{DF}, \overline{AB} \cong \overline{DE}$

Prove: $\overline{BC} \cong \overline{EF}$

AC ≅ DF, AB ≅ DE (given)
AC = DF, AB = DE (def ≅ segments)
AC – AB = DF – DE (subtraction =)
AC = AB + BC, DF = DE + EF (segment addition post)
AC – AB = BC, DF – DE = EF (subtraction =)
DF – DE = BC (substitution =)
BC = EF (substitution =)
BC ≅ EF (def ≅ segments)

2.5 PROVING STATEMENTS ABOUT SEGMENTS AND ANGLES

- Prove this property of angle bisectors: If you know MP bisects $\angle LMN$, prove that two times $m\angle LMP$ is $m\angle LMN$.
- **Given:** $\overrightarrow{MP}$ bisects $\angle LMN$
- **Prove:** $2(m\angle LMP) = m\angle LMN$



- 99 #1, 2, 4, 6, 10, 12, 14, 16, 17, 18, 23, 24, 25, 27, 30

STATEMENTS	**REASONS**
1. $\overrightarrow{MP}$ bisects $\angle LMN$.	**1.** Given
2. $\angle LMP \cong \angle NMP$	**2.** Definition of angle bisector
3. $m\angle LMP = m\angle NMP$	**3.** Definition of congruent angles
4. $m\angle LMP + m\angle NMP = m\angle LMN$	**4.** Angle Addition Postulate
5. $m\angle LMP + m\angle LMP = m\angle LMN$	**5.** Substitution Property of Equality
6. $2(m\angle LMP) = m\angle LMN$	**6.** Distributive Property

2.6 PROVING GEOMETRIC RELATIONSHIPS

Objectives: By the end of the lesson,

- I can prove geometric relationships by writing fl owchart proofs.
- I can prove geometric relationships by writing paragraph proofs.

2.6 PROVING GEOMETRIC RELATIONSHIPS

All right angles are congruent

Congruent Supplements Theorem

If two angles are supplementary to the same angle (or to congruent angles), then they are congruent

Congruent Complements Theorem

If two angles are complementary to the same angle (or to congruent angles), then they are congruent

2.6 PROVING GEOMETRIC RELATIONSHIPS

Linear Pair Postulate

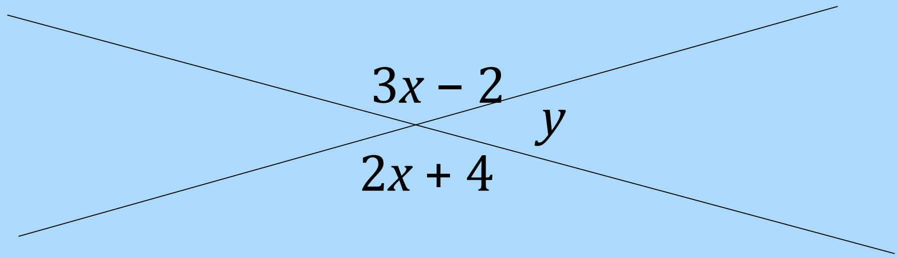
If two angles form a linear pair, then they are supplementary

Vertical Angles Congruence Theorem

Vertical angles are congruent

2.6 PROVING GEOMETRIC RELATIONSHIPS

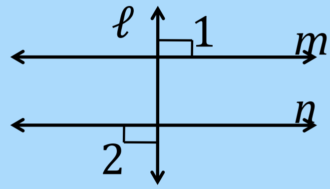
- Find x and y



$3x – 2 = 2x + 4 \rightarrow x = 6$
$y = 180 – (3x – 2) = 180 – (3(6) + 4) = 180 – (18 + 4) = 180 – 22 = 158$

2.6 PROVING GEOMETRIC RELATIONSHIPS

- Given: $\ell \perp m, \ell \perp n$
- Prove: $\angle 1 \cong \angle 2$



Statements	Reasons

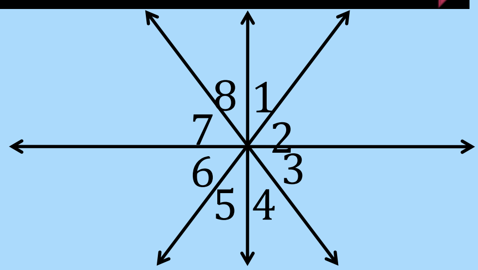
$\ell \perp m, \ell \perp n$ (given)

$\angle 1$ and $\angle 2$ are right angles (def of $\perp$ lines)

$\angle 1 \cong \angle 2$ (All rt $\angle$'s are $\cong$)

2.6 PROVING GEOMETRIC RELATIONSHIPS

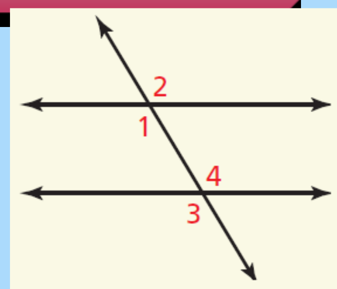
- Write a paragraph proof.
- Given:
  - $\angle 1$ and $\angle 3$ are complements
  - $\angle 3$ and $\angle 5$ are complements
- Prove: $\angle 1 \cong \angle 5$



We are given $\angle 1$ and $\angle 3$ are complements and $\angle 3$ and $\angle 5$ are complement. By the congruent complements theorem $\angle 1 \cong \angle 5$.

2.6 PROVING GEOMETRIC RELATIONSHIPS

- Write a flow proof.
- **Given** $\angle 1 \cong \angle 4$
- **Prove** $\angle 2 \cong \angle 3$



Statement	Reason
$\angle 1 \cong \angle 4$	Given
$\angle 2 \cong \angle 1$	Vert. ∠'s ≅
$\angle 4 \cong \angle 3$	Vert. ∠'s ≅
$\angle 2 \cong \angle 4$	Transitive
$\angle 2 \cong \angle 3$	Transitive

107 #2, 4, 6, 8, 10, 12, 13, 15, 17, 18, 19, 24, 26, 29, 31